Title: It's a Wrap!

Brief Overview:

The following activities are designed to incorporate mathematics, the arts, and language arts into a project which introduces students to the concepts of design and engineering by applying measurement, estimation, and computational skills. The students will calculate the surface area and volume of boxes and bags. They will design and select materials for a box or bag using a net. They also will communicate mathematical methods and ideas. Students will make a Language Arts connection by creating a jingle for their cookies. They will make an Art connection by decorating their packages.

NCTM 2000 Principles for School Mathematics:

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*
- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*
- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*
- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*
- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*
- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

Links to NCTM 2000 Standards:

- **Content Standards**

Number and Operations
Students will use computational tools and strategies fluently and estimate appropriately.

Geometry
Students will use visualization and spatial reasoning to solve problems both within and outside mathematics.

Measurement
Students will understand attributes, units, and systems of measurement. They will apply a variety of techniques, tools, and formulas for determining measurement.

Data Analysis and Probability
Students will pose questions and select, organize, and represent data to answer questions.

- **Process Standards**

  <u>Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation</u>
  These five process standards are threads that integrate throughout the unit, although they may not be specifically addressed in the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

  Students will build new mathematical knowledge through their work with problems. They will select and use various types of reasoning and methods of mathematical thinking to communicate with others; express mathematical ideas coherently and clearly to their teacher; and use the language of mathematics as a precise means of mathematical expression. Furthermore, they will recognize and use connections among different mathematical ideas; and recognize, use, and learn about mathematics in contexts outside of mathematics. Last of all, students will create and use representations to organize, record, and communicate mathematical ideas.

Links to MSDE Writing Learning Outcomes:

- **Writing to Inform**
  Students will demonstrate the ability to write effectively to inform by justifying their choices for materials used in their product.

- **Writing for Personal Expression**
  Students will demonstrate the ability to write effectively to express personal ideas by writing an advertising jingle. In this way, they will create meaning using personal and fictional ideas.

Grade/Level:

Grades 6 - 8

Duration/Length:

Six (6) - 45 minute mathematics periods

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Measuring with a ruler in centimeters
- Calculating surface area and volume using formulas for cylinders and rectangular prisms
- Adding and multiplying decimals
- Using nets for three dimensional figures

Student Outcomes:

Students will:

- measure an open package to the nearest centimeter.
- analyze data and make decisions on their chosen materials
- construct a three dimensional package using the chosen material.
- design a net for the three dimensional package.
- write to inform.

- write for personal expression.

Materials/Resources/Printed Materials:

For each group:

- Rulers
- Calculators
- Colored markers
- Centimeter grid paper
- Poster board
- Foil
- Plastic Wrap
- Wax Paper
- Scissors, Tape, Glue
- Samples of boxes and bags used in packaging
- Worksheets

Development/Procedures:

- **Day One**
  Introduce the lesson with a discussion of food product packaging. Ask students to comment particularly on cookie packaging. Announce to the class that Nabasco is sponsoring a contest to design a package for "Coreo" cookies. The winner will receive a lifetime supply of "Coreo" cookies! To win the contest, the students must maximize the volume of their package, minimize wasted space, and minimize the cost of packaging. Refer to, "It's a Wrap! An Engineering Project" sheet.

  Students will be arranged in groups. As a group, they will examine several packages for store-bought cookies. They will be asked to note the size of the package, the material used for the package, the surface area of the package, and the volume of the package. They will also note the volume and weight of the cookies. Refer to Worksheet 1.

  Homework Assignment: Do the activity on the last page of the Examples and Commonly Asked Questions Worksheet.

- **Day Two**
  Students will be given information about the cookies they will be packaging (Handout). Based on the shape, size, and thickness, they will brainstorm ideas for packaging their product and keep a record of the possibilities on Worksheet 2. By the end of the class period each group should know what type of packaging they will use.

- **Day Three**
  Students will draw a net of their package to scale using centimeter grid paper (Worksheet 3). They will label the net with the correct dimensions and begin assembly of their actual package.

- **Day Four**
  Students will be given the nutritional information worksheet to determine the nutritional content of their package. They will affix this information to their finished package. They will complete assembling and decorating their package. (Worksheet 4)

- **Day Five**
  Students will compose their jingles for the marketing campaign. (Worksheet 5)

Homework Assignment: Complete Worksheet 6, "Discussion and Evaluation". This will be discussed in class tomorrow.

- **Day Six**
  Students will present their packages to their peers and the judging will take place. Discuss and evaluate the activity according to the guidelines on Worksheet 6.

Performance Assessment:

Students will be assessed daily based on performance. Both computational ability and completed tasks will be used to evaluate each student's progress. Scoring rubrics and a grading sheet for this activity are included in the attached pages.

Extension/Follow Up:

Students can compare the volume of cookies in commercial packages to the volume of the packaging. They can estimate the cost of packaging and the cost of cookies to estimate the profit a company is making.

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IT'S A WRAP
AN ENGINEERING PROJECT

The Nabasco Corporation (no relation to Nabisco) is sponsoring a contest to find a new package for its "Coreo" cookies (no relation to Oreo!). The winning team will win a lifetime supply of "Coreos"! The design constraints for the package are as follows:

1) The package must hold 24 "Coreos" (double-stuffed, not regular). **Each Coreo is 1.5 cm thick with a diameter of 5 cm.**

2) The package must be in the shape of a cylinder, box, or a bag (for easy supermarket stackability and display).

3) The package material must be one of or a combination of the following:

Foil:	**$.0003/cm^2**
Plastic	**$.0005/cm^2**
Wax paper	**$.0001/cm^2**
Paper	**$.00015/cm^2**
Poster board	**$.0002/cm^2**

4) The package material must maximize the freshness of the cookies.

5) The cost of the package must be minimized.

6) Your team must submit the calculations you used to find the surface area and volume of your chosen design for Nabasco's engineers to use as blueprints (Worksheet 2).

7) **Your team must construct a full scale prototype of your package (Worksheet 3).**

8) The nutritional information per cookie, required by law, must be given (Worksheet 4).

9) Your team must write a jingle that Nabasco's advertising company can use to market and increase the sales of the newly packaged (Worksheet 5).

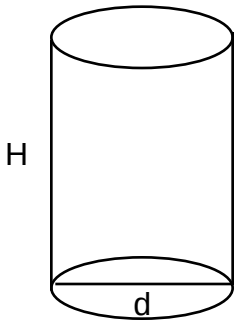
10) Your team must write a statement which explains / justifies your choice of material and the shape and size of your package, to be presented with the prototype (Worksheet 2).

IT'S A WRAP
EXAMPLES AND COMMONLY ASKED QUESTIONS

1) **What units should our team work in?**

All calculations and final answers should be done in centimeters and rounded to the nearest hundredth (two decimal places).

2) **How do we find the volume of a cylinder?**



The volume of a cylinder is given by the formula:

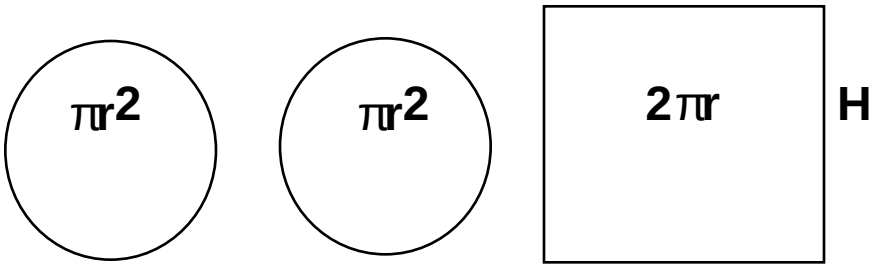
$V = \pi * r^2 * H$ (r is the radius or one half of the diameter, d)
(H is the height of the cylinder)

If r = 3 cm and H = 10 cm, the volume would be:

$V = \pi * (3\text{ cm})^2 * (10\text{ cm}) = \underline{282.74\text{ cm}^3}$ (use the π key on your calculator)

3) **How do we find the surface area of a cylinder?**

A cylinder can be broken up like this:



Surface Area = $\pi r^2 + \pi r^2 + (2\pi r)*(H)$ **($2\pi r$ is the circumference of the cylinder, and is also the length of the rectangle - think of unwrapping a soup can label)**

Using the same r and H,

S.A. = $(\pi * 3^{2)} + (\pi * 3^{2)} + (2*\pi*3)(10) = 9 * \pi + 9 * \pi + 60 * \pi = 78 * \pi =$ $\underline{245.04\ cm^2}$

4) **If our group chose to make a cylindrical package out of foil, how much would it cost?**

$SA_{cyl} = 245.04\ cm^2$

foil = \$.0003 / cm^2

Cost = 245.04 cm^2 * \$.0003 / cm^2 = \$.07 per package

5) **Can this example cylinder hold 24 "Coreos"?**

The volume of one "Coreo" (it is a cylinder - sketch it here) with a radius of 2.5 cm and a height of 1.5 cm can be found by:

$V_{Coreo} = \pi * (2.5\ cm)^2 * (1.5\ cm) = \pi * (6.25\ cm^2) * (1.5cm) = \underline{29.45\ cm^3}$

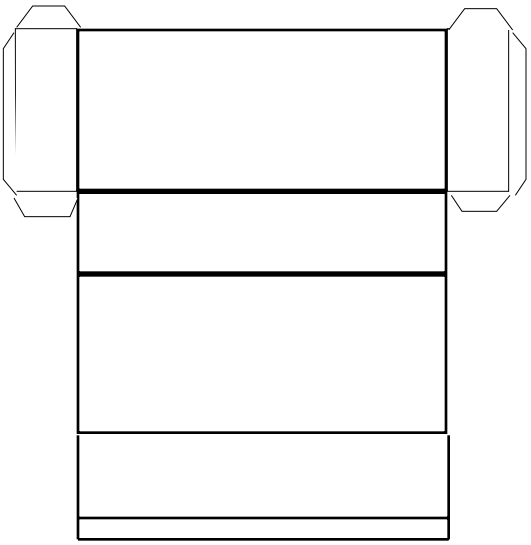
The volume of the cylinder was 282.74 cm^3

The number of cookies this cylinder can hold is found by:

282.74 cm^3 / 29.45 cm^3 = 9.6 cookies or 9 whole ones.
This means that this cylinder would NOT have enough space to hold 24 cookies.

6) What is a net?

A net is a flat, two-dimensional representation of a three-dimensional geometric solid. An example is shown below.



What shape would the three-dimensional view of this net produce?

Your team will be using one-centimeter graph paper to design the net for your package (each side of every square is one centimeter long). The net of your "actual" package will probably be too large to draw on graph paper; the solution to this problem is to draw the package "to scale", just as architects do when they make blueprints for houses.

In this project, the scale you will use is 1 cm = .5 cm.

This means that every "real-life" centimeter will be drawn as .5 centimeters. An actual length of 20 centimeters would therefore be drawn as 10 centimeter squares on your graph paper.

Use a centimeter ruler to measure the dimensions of the net above. What are its "real-life", or actual dimensions?

For homework, find the volume and surface area of a box with these dimensions:

length = 3 cm
width = 3 cm
height = 10 cm (Sketch a 3-D view of the box first; this will help you visualize the problem)

If the box is made of poster board, how much would it cost? Is there enough space inside of it to hold 24 "Coreos"? Mathematically show why or why not.

IT'S A WRAP
PROJECT CHECKLIST

Our team has:

- ☐ Selected package material(s).
- ☐ Completed calculation for finding the cost of the package material.
- ☐ Drawn a net of the chosen package in a 1 cm = .5 cm scale.
- ☐ Completed surface area calculations for package shape.
- ☐ Completed volume calculations for package shape.
- ☐ Completed math conversions to create an accurate nutritional guide.
- ☐ Constructed prototype of package with decoration and nutritional guide.
- ☐ Written a jingle to advertise the new product.
- ☐ Completed the evaluation questions.

Worksheet One

Using the three sample packages (box, bag, and cylinder), sketch a net (including tabs) for each package. Include dimensions on your sketches.

Box Bag Cylinder

Measure each package (in centimeters) and calculate total Surface Area and Volume for each package (estimate tabs).

Surface Area _____ **Surface Area** _____ **Surface Area** _____

Volume _____ **Volume** _____ **Volume** _____

(See Handout for formulas.)

Worksheet Two

Sketch your package and label with the actual dimensions of your package (include tabs).

One “Coreo” has a radius of 2.5 cm and a height of 1.5 cm . Find the following (refer to the Handout for the costs of materials and to the Examples for help):

Volume of one cookie: _____
Volume of twenty-four cookies: _____
Volume of package: _____

Surface area of package: _____
Cost of packaging (Surface area times cost of material): _____

Use the above information to justify your choice of material, shape, and size.

_____

_____

_____

_____

_____

_____

_____

_____

Worksheet Three

Draw a net of your package to scale using the centimeter grid below. Label the net with the correct dimensions (include tabs).

Worksheet Four

Based on the information given on the left side below for a serving size of 3 cookies, fill in the blanks on the right side below for a serving size of 1 cookie. The completed information on the right side should be included on the newly designed packaging that you create.

Nutrition Facts

Serving Size 3 cookies (33 g)

Servings Per Container - 17

Total Fat 7

Cholesterol 0 mg

Sodium 220 mg

Total Carbohydrate 23 g

- Dietary Fiber 1 g
- Sugars 13 g

Protein 1 g

Nutrition Facts

Serving Size 1 cookie (_____ g)

Servings Per Container - _____

Total Fat _____ g

Cholesterol _____ mg

Sodium _____ mg

Total Carbohydrate _____ g

- Dietary Fiber _____ g
- Sugars _____ g

Protein _____ g

Worksheet Five

Compose a jingle or short song that will help to market your product. It should have a minimum of four lines and should include the name “Coreo Cookies”. Remember, the purpose of the jingle is to help the consumer identify with the jingle so that he/she will purchase the product.

_____

_____

_____

_____

_____

_____

_____

_____

Worksheet Six

Discussion and Evaluation

1. What problems did you encounter doing this project? Be specific.

2. This project is an example of an engineering project. What did you learn about engineering from this project?

3. On a scale of 1 to 5, with 5 being the highest, how would you rate your group's skills as problem solvers? Why?

4. To what extent did all members of the group participate?

5. How did this project increase your understanding of surface area and volume?

6. Would you be able to determine the surface area and volume of a triangular prism? Explain the steps.

Rubric for Scoring
Mathematical Content

3 Points -

- Student fully participated in each activity each day.
- Student demonstrated complete understanding of addition, subtraction, multiplication, and division.
- Student measured accurately on Worksheet 1 (examples and design).
- Student made accurate calculations of Volume and Surface Area on Worksheet 2.
- Student created an accurate net for the packaging. Net was labeled accurately.
- Students used ratio and proportion to determine the correct nutritional value on Worksheet 3.
- Student assembled the package, decorated the package, and attached the nutritional label correctly.

2 Points -

- Student participated in most of the activities.
- Student demonstrated some understanding of addition, subtraction, multiplication, and division.
- Student made only minimal errors in measurement on Worksheet 1.
- Student made minimal errors on Worksheet 2.
- Student made minimal errors in creating the net. Net was not labeled.
- Student demonstrated some understanding of ratio and proportion on Worksheet 3.
- Student participated minimally in the assembly of the packaging.

1 Point -

- Student participated minimally in the activities.
- Student showed limited understanding of addition, subtraction, multiplication, and division.
- Student made many errors in measurement on Worksheet 1.
- Student made many errors on Worksheet 2.
- Student made many errors in creating the net.
- Student demonstrated limited understanding of ratio and proportion on Worksheet 3.
- Student did not participate in the package assembly.

IT'S A WRAP: AN ENGINEERING PROJECT
GRADING SHEET

Group Members 1._____

2._____

3._____

Worksheet 1	Complete	3	2	1	0
	Accurate	3	2	1	0
Worksheet 2	Complete	3	2	1	0
	Accurate	3	2	1	0
Worksheet 3	Complete	3	2	1	0
	Accurate	3	2	1	0
Worksheet 4	Complete	3	2	1	0
	Accurate	3	2	1	0
Worksheet 5	Complete	3	2	1	0
Worksheet 6	Complete	3	2	1	0

Presentation:

Price of Packaging	$0.05 - $0.15	3 points			
	$0.16 - $0.25	2 points			
	$0.26 - $0.35	1 points			
	$0.36 - above	0 points			
Quality of Packaging	Sturdiness	3	2	1	0
	Stackability	3	2	1	0
	Freshness	3	2	1	0
Design of Package	Colorful	3	2	1	0
	Attractive	3	2	1	0
	Nutrition Label	3	2	1	0
Marketing Campaign		3	2	1	0

Possible Points: 54 **Total Points:_____ = _____%**

Rubric for Scoring
Language Arts Content

3 Points-

- Student fully participated in each activity each day.
- Student demonstrated complete understanding of the following skills: punctuation, capitalization, grammatical usage, and syntax.
- Student accurately completed Worksheet 4 with given requirements.

2 Points-

- Student participated in most of the activities.
- Student demonstrated some understanding of punctuation, capitalization, grammatical usage, and syntax.
- Student completed Worksheet 4 with minimal errors.

1 Point-

- Student participated minimally in the activities.
- Student showed limited understanding of required grammatical knowledge.
- Student made many errors on Worksheet 4 or did not complete.